

COMPUTATION OF AREA AND VOLUME

Aim:

One of the main objectives of the surveying is to compute the areas and volumes. Generally, the lands will be of irregular shaped polygons. There are formulae readily available for regular polygons like, triangle, rectangle, square and other polygons. But for determining the areas of irregular polygons, different methods are used.

They are:

- (1) Graphical method
- (2) Co-ordinate method
- (3) Planimeter

Out of these three methods, the co-ordinate method is popularly used, in land surveying for computing catchment area, drainage area, cross section of rivers, channels etc. Under this method the given area is split into two with a base line run at the centre. There are two important rules available.

1. Trapezoidal Rule

In this method, boundaries between the ends of ordinates are assumed to be straight. Thus the area enclosed between these line and the irregular boundary lines are considered as trapezoids.

$$A = \frac{d}{2} [O_1 + O_n + 2(O_2 + O_3 + O_4 + \dots + O_{n-1})]$$

A = distance between ordinate/ 2*[{first ordinate + last ordinate} + 2{sum of other ordinates}]

2. Simpson's Rule:

$$A = \frac{d}{3} [O_1 + O_n + 4(O_2 + O_4 + \dots) + 2(O_3 + O_5 + \dots)]$$

$$A = \frac{\text{Common distance (d)}}{3} \left[(\text{First ordinate} + \text{Last ordinate}) + 4(\text{Sum of even ordinates}) + 2(\text{Sum of odd ordinates}) \right]$$

Limitations:

The rule is applicable only when the number of divisions is even or the number of ordinates are odd sometimes one or both end ordinates may be zero. However they must be taken into account while applying rules.

WORKOUT PROBLEMS

1. The following offsets were taken from a chain line to an irregular boundary line at an interval of 10 m. 0, 2.50, 3.50, 5.00, 4.60, 3.20, 0 m. Compute the area between the chain line, the irregular boundary line and the end offsets by:

(a) Trapezoidal Rule

(b) Simpson’s Rule

(a) Trapezoidal Rule

Here d = 10

$$\text{Area} = \frac{10}{2} \{0 + 0 + 2(2.50 + 3.50 + 5.00 + 4.60 + 3.20)\} = 5 * 37.60 = \mathbf{188 \text{ m}^2}$$

(b) Simpson’s Rule

D = 10

$$\text{Area} = \frac{10}{3} \{0 + 0 + 4(2.50 + 5.00 + 3.20) + 2(3.50 + 4.60)\} = \frac{10}{3} * 59.00 = \mathbf{196.66 \text{ m}^2}$$

2. The following offsets were taken from a survey line to a curved boundary line:

Distance (m)	0	5	10	15	20	30	40	60	80
Offset (m)	2.50	3.80	4.60	5.20	6.10	4.70	5.80	3.90	2.20

Find the area between the survey line, the curved boundary line and the first and last offsets by (a) Trapezoidal Rule and (b) Simpson’s Rule.

Here, the intervals between the offsets are not regular throughout the length. Soothe section is divided into three compartments.

Let,

$$\Delta_1 = \text{Area of the 1}^{\text{st}} \text{ section}$$

$$\Delta_2 = \text{Are of the 2}^{\text{nd}} \text{ section}$$

$$\Delta_3 = \text{Area of the 3}^{\text{rd}} \text{ section}$$

Here,

$$d_1 = 5 \text{ m}$$

$$d_2 = 10 \text{ m}$$

$$d_3 = 20 \text{ m}$$

(a) Trapezoidal Rule:

$$\Delta_1 = \frac{5}{2} \{2.50 + 6.10 + 2(3.80 + 4.60 + 5.20)\} = 89.50 \text{ m}^2$$

$$\Delta_2 = \frac{10}{2} \{6.10 + 5.80 + 2(4.70)\} = 106.50 \text{ m}^2$$

$$\Delta_3 = \frac{20}{2} \{5.80 + 2.20 + 2(3.90)\} = 158.00 \text{ m}^2$$

$$\text{Total Area} = 89.50 + 106.50 + 158.00 = \mathbf{354.00 \text{ m}^2}$$

(b) By Simpson's Rule

$$\Delta_1 = \frac{5}{3} \{2.50 + 6.10 + 4(3.80 + 5.20) + 2(4.60)\} = 89.66 \text{ m}^2$$

$$\Delta_2 = \frac{10}{3} \{6.10 + 5.80 + 4(4.70)\} = 102.33 \text{ m}^2$$

$$\Delta_3 = \frac{20}{3} \{5.80 + 2.20 + 4(3.90)\} = 157.33 \text{ m}^2$$

$$\text{Total area} = 89.66 + 102.33 + 157.33 = \mathbf{349.32 \text{ m}^2}$$

EXERCISE

The following offsets were taken at 15 m intervals from a survey line to an irregular boundary line. 3.50, 4.30, 6.75, 5.25, 7.50, 8.80, 7.90, 6.40, 4.40, 3.25 m. Calculate the area enclosed between the survey line, the irregular boundary line and the first and last offsets by:

(a) Trapezoidal Rule

(b) Simpson's Rule

COMPUTATION OF VOLUMES

The computation of volumes of various quantities from the measurements done in the field is required in the design and planning on many engineering works. The volume of earth work is required for suitable alignment of road works, canal and sewer lines, soil and water conservation works, farm pond and percolation pond consent.

The computation of volume of various materials such as coal, gravel and is required to check the stock files, volume computations are also required for estimation of capacities of bins tanks etc.

For estimation of volume of earth work cross sections are taken at right angles to a fixed line, which runs continuously through the earth work. The spacing of the cross sections will depend upon the accuracy required. The volume of earth work is computed once the various cross-sections are known, adopting Prismoidal rule and trapezoidal rule.

Trapezoidal rule : $V = \frac{D}{2} [A_1 + A_n + 2(A_2 + A_3 + \dots + A_{n-1})]$

Prismoidal rule : $V = \frac{D}{3} [A_1 + A_n + 4(A_2 + A_4 + \dots) + 2(A_3 + A_5 + \dots)]$

Where,

D - common distance between sections

A_1, A_2, \dots, A_n = cross sectional areas

WORKOUT PROBLEMS

1. Compute the cost of earth work involved in cutting open a trench of following size. Length 200 m, side slope 2: 1, depth of trench 4 m, bottom, width of trench 1.5 m. Cost of earth work Rs. 50 per m³.

$$\begin{aligned} \text{Cross sectional area of trench, } A &= (b + sh) * h \\ A &= (1.5 + 2*4) * 4 \\ A &= 9.5 * 4 = 38 \text{ m}^2 \end{aligned}$$

∴ Volume of earth work, $V = A * L = 38 * 200 = 7600 \text{ m}^3$

∴ Cost of earth work = $7600 * 50 = \text{Rs. } 3,80,000.00$

2. Compute the volume of earth work involved in constructing a farm pond of the following size: size, at bottom 6 x 4 m. Side slope 2: 1, depth of pond 4 m work out the cost of earth work also if it costs Rs. 50 per m³.

$$\text{Size of pond at bottom} = 6 \times 4 \text{ m}$$

$$\text{Area at bottom} = 24 \text{ m}^2 (a_1)$$

Size of pond at ground level:

$$\text{Length of pond} = 6 + 8 + 8 = 22 \text{ m}$$

$$\text{Breadth of pond} = 4 + 8 + 8 = 20 \text{ m}$$

$$\text{Cross sectional area of pond at ground level} = 20 * 22 = 440 \text{ m}^2 (a_3)$$

$$\text{Area of pond at mid height} = \frac{(22+6)}{2} * \frac{(20+4)}{2} = 14 * 12 = 168 \text{ m}^2 (a_2)$$

$$\text{Using prismoidal rule, } V = \frac{D}{2} [a_2 + a_3 + 2(a_1)]$$

$$V = \frac{D}{2} [24 + 440 + 2(168)]$$

$$V = \frac{2}{2} [464 + 336] = 800 \text{ m}^3$$

$$\therefore \text{Cost of earth work} = 50 * 800 = \text{Rs. 40,000}$$

EXERCISE

1. The three cross section of embankment at an interval of 30 m. Compute the volume of earth required to form the embankment?