

Practical.3

Measures of central tendency – mean, median, mode, geometric mean and harmonic mean for grouped data

Arithmetic mean or mean

Grouped Data

The mean for grouped data is obtained from the following formula:

$$\bar{x} = \frac{\sum fx}{N}$$

Where x = the mid-point of individual class

f = the frequency of individual class

N = the sum of the frequencies or total frequencies.

Short-cut method

$$\bar{x} = A + \frac{\sum fd}{N} \times c$$

Where $d = \frac{x - A}{c}$

A = any value in x

N = total frequency

c = width of the class interval

Example 1

Given the following frequency distribution, calculate the arithmetic mean

Marks	:	64	63	62	61	60	59
Number of	}						
Students		:	8	18	12	9	7

Solution

X	f	fx	d=x-A	fd
64	8	512	2	16
63	18	1134	1	18
62	12	744	0	0
61	9	549	-1	-9
60	7	420	-2	-14
59	6	354	-3	-18
	60	3713		-7

Direct method

$$\bar{x} = \frac{\sum fx}{N}$$

$$\bar{x} = \frac{3713}{60} = 61.88$$

Short-cut method

$$\bar{x} = A + \frac{\sum fd}{N} \times c$$

Here A = 62

$$\bar{x} = 62 - \frac{7}{60} \times 1 = 61.88$$

Example 2

For the frequency distribution of seed yield of sesamum given in table calculate the mean yield per plot.

Yield per plot in(in g)	64.5-84.5	84.5-104.5	104.5-124.5	124.5-144.5
No of plots	3	5	7	20

Solution

Yield (in g)	No of Plots (f)	Mid X	$d = \frac{x - A}{c}$	fd
64.5-84.5	3	74.5	-1	-3
84.5-104.5	5	94.5	0	0
104.5-124.5	7	114.5	1	7
124.5-144.5	20	134.5	2	40
Total	35			44

$$A=94.5$$

The mean yield per plot is

$$\bar{x} = A + \frac{\sum fd}{N} \times c$$

$$\bar{x} = 94.5 + \frac{44}{35} \times 20 = 119.64 \text{ g}$$

Median

Grouped data

In a grouped distribution, values are associated with frequencies. Grouping can be in the form of a discrete frequency distribution or a continuous frequency distribution. Whatever may be the type of distribution, cumulative frequencies have to be calculated to know the total number of items.

Cumulative frequency: (cf)

Cumulative frequency of each class is the sum of the frequency of the class and the frequencies of the previous classes, ie adding the frequencies successively, so that the last cumulative frequency gives the total number of items.

Discrete Series

Step1: Find cumulative frequencies.

Step2: Find $\left(\frac{N}{2} + 1\right)$

Step3: See in the cumulative frequencies the value just greater than $\left(\frac{N}{2} + 1\right)$

Step4: Then the corresponding value of x is median.

Example 3

The following data pertains to the number of members in a family. Find median size of the family.

Number of members x	1	2	3	4	5	6	7	8	9	10	11	12
Frequency f	1	3	5	6	10	13	9	5	3	2	2	1

Solution

X	f	cf
1	1	1
2	3	4
3	5	9
4	6	15
5	10	25
6	13	38
7	9	47
8	5	52
9	3	55
10	2	57
11	2	59
12	1	60
	60	

$$\begin{aligned} \text{Median} &= \text{size of } \left(\frac{N+1}{2} \right)^{\text{th}} \text{ item} \\ &= \text{size of } \left(\frac{60+1}{2} \right)^{\text{th}} \text{ item} \\ &= 30.5^{\text{th}} \text{ item} \end{aligned}$$

The cumulative frequency just greater than 30.5 is 38. and the value of x corresponding to 38 is 6. Hence the median size is 6 members per family.

Continuous Series

The steps given below are followed for the calculation of median in continuous series.

Step1: Find cumulative frequencies.

Step2: Find $\left(\frac{N}{2} \right)$

Step3: See in the cumulative frequency the value first greater than $\left(\frac{N}{2} \right)$, Then the corresponding class interval is called the Median class. Then apply the formula

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c$$

Where l = Lower limit of the median class

m = cumulative frequency preceding the median

c = width of the median class

f = frequency in the median class.

N = Total frequency.

Example 4

For the frequency distribution of weights of sorghum ear-heads given in table below. Calculate the median.

Weights of ear heads (in g)	No of ear heads (f)	Cumulative frequency (m)
60-80	22	22
80-100	38	60
100-120	45	105
120-140	35	140
140-160	20	160
Total	160	

Solution

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c$$

$$\left(\frac{N}{2}\right) = \left(\frac{160}{2}\right) = 80$$

Here $l = 100$, $N=160$, $f = 45$, $c = 20$, $m = 60$

$$\text{Median} = 100 + \frac{80 - 60}{45} \times 20 = 108.8 \text{ gms}$$

Geometric mean

Grouped Data

For grouped data

$$\text{GM} = \text{Antilog} \left[\frac{\sum f \log x_i}{N} \right]$$

Example 5

Find the Geometric mean for the following

Weight of sorghum (x)	No. of ear head(f)
50	4
65	6
75	16
80	8
95	7
100	4

Solution

Weight of sorghum (x)	No. of ear head(f)	Log x	flog x
50	5	1.699	8.495
63	10	10.799	17.99
65	5	1.813	9.065
130	15	2.114	31.71
135	15	2.130	31.95
Total	50	9.555	99.21

Here N= 50

$$\begin{aligned}
 \text{GM} &= \text{Antilog} \left[\frac{\sum f \log x_i}{N} \right] \\
 &= \text{Antilog} \left[\frac{99.21}{50} \right] \\
 &= \text{Antilog } 1.9842 = 96.43
 \end{aligned}$$

Continuous distribution

Example 6

For the frequency distribution of weights of sorghum ear-heads given in table below. Calculate the Geometric mean

Weights of ear heads (in g)	No of ear heads (f)
60-80	22
80-100	38
100-120	45
120-140	35
140-160	20
Total	160

Solution

Weights of ear heads (in g)	No of ear heads (f)	Mid x	Log x	f log x
60-80	22	70	1.845	40 59
80-100	38	90	1.954	74.25
100-120	45	110	2.041	91.85
120-140	35	130	2.114	73.99
140-160	20	150	2.176	43.52
Total	160			324.2

Here N = 160

$$GM = \text{Antilog} \left[\frac{\sum f \log x_i}{N} \right]$$

$$= \text{Antilog} \left[\frac{324.2}{160} \right]$$

$$= \text{Antilog} [2.02625]$$

$$= 106.23$$

Harmonic mean

For a frequency distribution

$$H.M = \frac{N}{\sum_{i=1}^n f\left(\frac{1}{x_i}\right)}$$

Example 7

The marks secured by some students of a class are given below. Calculate the harmonic mean.

Marks	20	21	22	23	24	25
Number of Students	4	2	7	1	3	1

Solution

Marks X	No of Students f	$\frac{1}{x}$	$f\left(\frac{1}{x}\right)$
20	4	0.0500	0.2000
21	2	0.0476	0.0952
22	7	0.0454	0.3178
23	1	0.0435	0.0435
24	3	0.0417	0.1251
25	1	0.0400	0.0400
	18		0.8216

$$H.M = \frac{N}{\sum f\left(\frac{1}{x_i}\right)} = \frac{18}{0.1968} = 21.91$$

Learning Exercise

For the following frequency distribution find the

- (i) Mean
- (ii) Median
- (iii) Mode
- (iv) Harmonic mean
- (iv) Geometric mean

Weight of earheads in gms	No. of earhead
40 - 60	6
60 - 80	8
80 - 100	35
100 - 120	55
120 - 140	30
140 - 160	15
160 - 180	12
180 - 200	9